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CALCULUS.

230. Proposed by C. N. SCHMALL, College of the City of New York.

The greatest rectangle is inscribed in an ellipse, and the greatest ellipse in that rectangle, again the greatest rectangle in that (second) ellipse, and the greatest ellipse in that (second) rectangle, and so on *ad infinitum*; show that the sum of all the inscribed rectangles is equal to the area of the rectangle circumscribed about the given ellipse.

Solution by J. E. SANDERS, Reinersville, Ohio, and A. H. HOLMES, Brunswick, Maine.

Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, be the equation of the ellipse. Let $2x_1$ be the length of its maximum inscribed rectangle. Then $2y_1 = 2\sqrt{(a^2b^2 - b^2x_1^2)} =$ its width, and the area of the rectangle is $4x_1y_1 = 4bx_1\sqrt{(a^2 - x_1^2)}/a =$ maximum.

$\therefore 4b\sqrt{(a^2 - x_1^2)} - \frac{4bx_1^2}{\sqrt{(a^2 - x_1^2)}} = 0$; from which $x_1 = \frac{1}{2}\sqrt{2}a = a_1$. Hence, $y_1 = \frac{1}{2}\sqrt{2}b = b_1$. Hence its area $= 4a_1b_1 = 2ab$. The equation of the ellipse inscribed in this rectangle is $\frac{x^2}{a_1^2} + \frac{y^2}{b_1^2} = 1$. Hence, the length of the maximum rectangle inscribed in this ellipse is $2(\frac{1}{2}\sqrt{2}a_1) = 2a_2$, its width is $2(\frac{1}{2}\sqrt{2}b_1) = 2b_2$, and its area is $4a_2b_2 = 2a_1b_1 = ab$. By induction, the area of the n th rectangle $= \frac{1}{2}$ of the area of the $(n-1)$ th rectangle, or $a_nb_n = \frac{1}{2}a_{n-1}b_{n-1}$. Hence, the sum of all the rectangles is $S = 2ab + ab + \frac{1}{2}ab + \frac{1}{4}ab + \frac{1}{8}ab + \dots$ *ad infinitum* $= 4ab$, which is the area of the rectangle circumscribing the original ellipse.

Also solved by G. B. M. Zerr, J. Scheffer, and G. W. Greenwood.

MECHANICS.

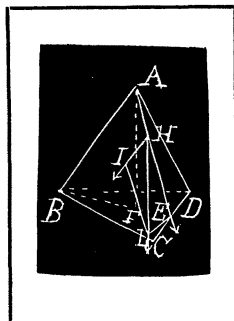
193. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

Three light smoothly jointed rods stand like a tripod—the three edges of a regular tetrahedron. A rectangular board, weight w , stands on this like an easel. Find the thrust on the rod which does not touch the easel.

Solution by G. B. M. ZERR, Ph. D., Parsons, W. Va.

Let $AB = a$ be one of the equal rods jointed at A ; H the center of gravity of the board, weight w ; HL the direction of the weight w . Resolve HL into the components HE along the median AE , and HI parallel to AB . Then HI is the thrust required. Now $AE = BE = \frac{1}{2}a\sqrt{3}$. Draw AF perpendicular to BE . Then $BF = \frac{2}{3}BE = \frac{1}{3}a\sqrt{3}$. $EF = \frac{1}{3}a\sqrt{3}$.

$\cos BAE = \cos IHE = \frac{1}{3}\sqrt{3}$, $\sin IHE = \frac{1}{3}\sqrt{6}$.



$$\sin IHL = \frac{1}{3}\sqrt{3}, \cos IHL = \frac{1}{3}\sqrt{6}.$$

$$\sin EHL = \frac{1}{3}, \cos EHL = \frac{2}{3}\sqrt{2}.$$

$$HI : HL = \sin ILH : \sin HIL = \sin EHL : \sin IHE. \therefore HI : w = \frac{1}{3} : \frac{1}{3}\sqrt{6}$$

or $HI = w/\sqrt{6} = w\sqrt{6}/6$; $HE : w = \sin IHL : \sin HIL$; $HE : w = \frac{1}{3}\sqrt{3} : \frac{1}{3}\sqrt{6}$, or $HE = w/\sqrt{2} = w\sqrt{2}/2$.

Also solved by G. W. Greenwood.

AVERAGE AND PROBABILITY.

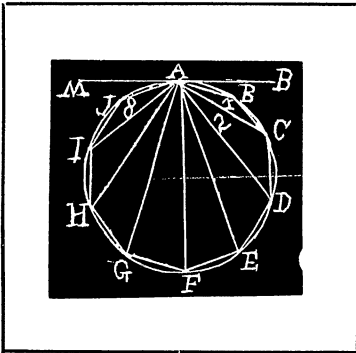
163. Proposed by R. D. CARMICHAEL, Anniston, Ala.

In a regular n -gon a triangle is formed by taking three vertices at random. What is the mean value of the triangle?

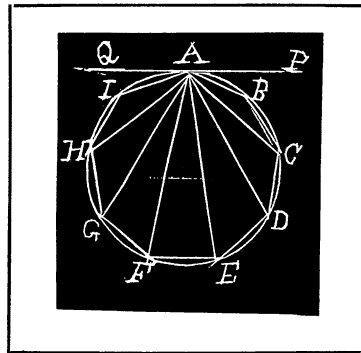
Solution by G. B. M. ZERR, Ph. D., Parsons, W. Va.

Whether n be even or odd, there can be formed, at any vertex, as B , or C , or D , etc., by joining it with A and any other vertex, $(n-2)$ triangles. Since there are n vertices, the total number of triangles $= n(n-2)$.

I. n even. For A the sum of the areas is the area of the polygon $= \frac{1}{2}nr^2 \sin(2\pi/n)$; for B and J combined $nr^2 \sin(2\pi/n) =$ sum of areas; for C and I , the sum is $2nr^2 \sin(2\pi/n) - 4(n-2)r^2 \sin(\pi/n) \sin(\pi/n) \sin(2\pi/n)$; for D and H , $3nr^2 \sin(2\pi/n) - 4nr^2 \sin(\pi/n) \sin(\pi/n) \sin(2\pi/n) - 4(n-4)r^2 \sin(\pi/n) \times \sin(2\pi/n) \sin(3\pi/n)$. For the next pair of vertices the sum is



n even.



n odd.

$$\begin{aligned} &4n^2 \sin(2\pi/n) - 4(n+2)r^2 \sin(\pi/n) \sin(\pi/n) \sin(2\pi/n) \\ &- 4(n-2)r^2 \sin(\pi/n) \sin(2\pi/n) \sin(3\pi/n) \\ &- 4(n-6)r^2 \sin(\pi/n) \sin(3\pi/n) \sin(4\pi/n). \end{aligned}$$

The sum of the areas of the triangles for $\frac{n-2}{2}$ th, the $\frac{n}{2}$ th and the $\frac{n+2}{2}$ th vertices are equal, and their combined sum for the three vertices is